# HEAT TRANSFER FROM ELECTRICALLY-HEATED THIN VERTICAL WIRES UNDER FREE CONVECTION

G. V. Mchedlidze

UDC 536.25

Results are given for an experimental study of the heat transfer from electrically-heated, thin, vertical resistive elements in a coaxial resistance standard (KRMS) in transformer oil under free convection.

Coaxial resistance standards of the KRMS type with standard frequency characteristics (1, 10, $10^2$, $10^3$ Ω) are intended for precise measurements of the frequency characteristics of standard resistors over a frequency range up to 20 kHz [1]. In this range the magnitude of the measured frequency correction of ac resistance standards does not exceed $5 \cdot 10^{-6}$ [2]. Elimination of the frequency-independent systematic temperature error during such measurements is one of the principal problems encountered because of the great difficulty of direct measurement of the temperature drop between the resistive element and the surrounding heat-absorbing medium during the dissipation of applied power. The resistive element of KRMS standards is a thin uninsulated vertical wire made of the high-impedance precision resistance alloys "Margalin" or "Terminal" having a temperature coefficient of resistance no greater than $\pm 3 \cdot 10^{-6}$ °K$^{-1}$ [1].

To achieve a measurement error of no more than $1 \cdot 10^{-6}$, the maximum amount of power dissipated in a standard resistor is $25 \cdot 10^{-3}$ W [3]. Assuming this value of the power to be a maximum also in the measurement of frequency characteristics of standard resistors, the need arises for the establishment of the heat-transfer coefficient $\alpha$ for each resistive element of a KRMS standard in order to determine the corresponding temperature drops or temperature differences $\Delta t$ between its surface and the surrounding heat-absorbing medium. Consequently, the applied power and the geometric dimensions of the KRMS are the only given parameters in this case.

The analytic solution of the problem is based on the determination of the heat-transfer conditions in a steady-state operating mode between a vertical wire and a large volume of heat-absorbing medium (the ratio between the diameters of the largest resistive element and the return wire of a KRMS is $1 \cdot 10^{-2}$ [1]). Since there is no forced motion of the heat-absorbing medium in the KRMS design, heat transfer will have the character of free convection. A photograph of a disassembled 1 Ω KRMS is shown in Fig. 1.

Heat transfer from electrically heated vertical wires under free convection in gases and liquids has been studied theoretically and experimentally [4]. Good confirmation was obtained experimentally for the following criterial heat-transfer equation for vertical wires and for a constant heat flow based on the theoretical studies [4]:

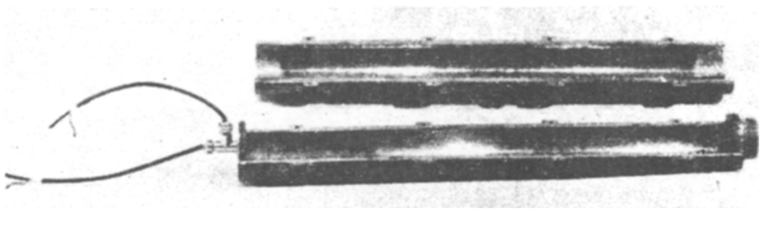

Fig. 1. View of disassembled 1 ΩKRMS (operating position is vertical).

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 2, pp. 233-237, February, 1974. Original article submitted July 25, 1973.

TABLE 1. Experimental Results for Heat Transfer from Vertical Thin Wires under Free Convection

| | Copper wire $d=30\cdot10^{-6}$ m; $l=25,5\cdot10^{-2}$ m | | | | | | Platinum wire $d=70\cdot10^{-6}$ m; $l=26\cdot10^{-2}$ m | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Expt. No | P, w/m$^2$ | q, w/m$^2$ | $\Delta t=t_n-t_M$, K | $\alpha$, w/m$^2$, °K | $Nu_d$ | $Ra_d d/l\ 10^{-10}$ | P, w • $10^{-6}$ | q, w/m$^2$ | $\Delta t=t_n-t_M$, K | $\alpha$, w/m$^2$, °K | $Nu_d$ | $Ra_d d/l\ 10^{-10}$ | $t_M$, K |
| 1 | 15818 | 658,5 | 0,46 | 1437,3 | 0,392 | 57,6 | 9331 | 163,3 | 0,32 | 511,3 | 0,325 | 1172,0 | |
| 2 | 15782 | 657,0 | 0,50 | 1309,6 | 0,357 | 63,1 | 9365 | 163,9 | 0,31 | 527,2 | 0,335 | 1141,5 | |
| 3 | 26575 | 1106,3 | 0,73 | 1515,2 | 0,413 | 91,8 | 21861 | 382,5 | 0,71 | 534,6 | 0,340 | 2626,9 | |
| 4 | 26541 | 1105,0 | 0,70 | 1566,9 | 0,427 | 88,6 | 21920 | 383,5 | 0,71 | 538,0 | 0,342 | 2617,2 | 293 |
| 5 | 39302 | 1636,2 | 1,11 | 1476,5 | 0,403 | 139,3 | 42436 | 742,6 | 1,32 | 564,1 | 0,359 | 4833,0 | |
| 6 | 39289 | 1635,7 | 1,11 | 1467,2 | 0,400 | 140,1 | 42481 | 743,3 | 1,19 | 623,2 | 0,396 | 4374,4 | |
| 1 | 16039 | 667,7 | 0,54 | 1242,3 | 0,345 | 108,0 | 9548 | 167,0 | 0,34 | 486,2 | 0,309 | 2016,2 | |
| 2 | 16033 | 667,5 | 0,49 | 1344,4 | 0,374 | 99,6 | 9541 | 167,0 | 0,33 | 499,3 | 0,317 | 1963,3 | |
| 3 | 26857 | 1118,1 | 0,81 | 1378,4 | 0,383 | 163,1 | 21898 | 383,2 | 0,64 | 594,1 | 0,378 | 3791,5 | 303 |
| 4 | 26833 | 1117,1 | 0,81 | 1377,5 | 0,383 | 162,9 | 22151 | 387,6 | 0,65 | 596,3 | 0,379 | 3820,9 | |
| 5 | 39559 | 1646,9 | 1,13 | 1460,0 | 0,406 | 227,0 | 34686 | 667,0 | 0,94 | 643,7 | 0,409 | 5543,2 | |
| 6 | 39569 | 1647,3 | 1,21 | 1356,9 | 0,377 | 243,9 | 34677 | 606,8 | 0,96 | 629,4 | 0,400 | 5666,7 | |

$$\mathrm{Nu}_d = 0.93\left(\mathrm{Ra}_d\frac{d}{l}\right)^{0.05}, \tag{1}$$

where $Ra_d = Gr_d Pr$ is the Rayleigh number.

However, experimental confirmation of Eq. (1) was given [4] only for the range $Ra_d d/l \geq 10^{-3}$ (for a wire with $d_{min} = 0.5$ mm). The values of $Ra_d d/l$ we have studied are less than $10^{-6}$, as is clear from Table 1. The diameters of the resistive elements in a KRMS are 0.366, 0.205, 0.065, and 0.025 mm [1].

Since there was no experimental verification in the literature of the analytically obtained criterial relation (1) in the range of interest to us, $Ra_d d/l \leq 10^{-6}$, we investigated the heat-transfer parameters for vertical wires of two diameters, 0.07 (platinum) and 0.03 mm (copper) installed in a KRMS structure in place of the usual resistive element for free convection in transformer oil with $t_m$ = 293 and 303 °K. Thus we covered experimentally the entire range of $Ra_d d/l$ characteristic of heat-transfer operating conditions for the resistive elements of all nominal KRMS standards [1].

In assembled form, a KRMS standard containing a test wire with high temperature coefficient of resistance (copper or platinum) was filled with transformer oil and set up vertically in a hermetically sealed brass cylinder which was also filled with transformer oil. For thermostatic control of this closed volume, a TS-24 thermostatic bath (working fluid, water) was used. Under the experimental conditions, the temperature of the water was maintained to the accuracy of ±0.05 °K and the temperature $t_m$ of the transformer oil in the closed volume away from the heated test wire was maintained to an accuracy of ±0.01 °K and was measured with a chromel—alumel thermocouple connected through a compensation circuit to a P-306 potentiometer.

The experiments were performed using the usually accepted technique [5] in which the test wire, electrically heated by a dc current, simultaneously served as a resistance thermometer. After constructing a calibration curve for the test wire, a stationary mode of heat transfer was established in each

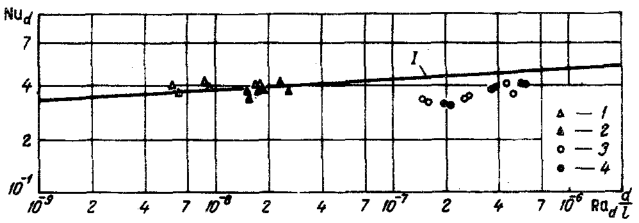


Fig. 2. Comparison of experimental data and criterial relation (1). Experimental data (transformer oil); 1) 0.03-mm wire, 3) 0.07-mm wire for $t_m$ = 293 °K; 2) 0.03-mm wire, 4) 0.07-mm wire for $t_m$ = 303 °K; I) $Nu_d = 0.93\ (Ra_d d/l)^{0.05}$.

experiment. In this case, the potential drop $U_0$ in the standard resistance $R_0$ and the potential drop $U_n$ in the test wire were measured. From the measured values of $U_0$ and $U_n$, values of $R_n$ were calculated for six values of the power load and the wire temperature $t_n$ for each experiment was determined from the calibration curve. The specific heat flux q at the surface of the test wire was calculated from these same values of $U_n$ and $R_n$.

Results of the experimental studies with the two test wires are given in Table 1.

A curve of the dependence of $Nu_d$ on $Ra_d d/l$ is plotted in Fig. 2 on a logarithmic scale for the values of $\alpha$ and $\Delta t$ obtained in each experiment. It is clear from the curve that the experimental data we obtained agrees satisfactorily with the theoretical curve based on the criterial relation (1).

The experimental results indicate that the criterial relation (1) is valid for the heat transfer from all resistive elements of KRMS standards. Consequently, the heat-transfer coefficients of the resistive elements and the values of the temperature drop $\Delta t$ corresponding to a given operating mode can be calculated from Eq. (1).

Since the desired quantity $\Delta t$ appears in both the left and right sides of Eq. (1) (on the left side, it is in the implicit form $\alpha = q/\Delta t$), Eq. (1) should be modified somewhat to obtain an expression for $\Delta t$. After simple transformations, we obtain the following computational expression:

$$\Delta t = \frac{q}{\left[0.93\,\frac{\lambda}{d}\left(\frac{\beta g d^3 q}{\nu^2}\,\mathrm{Pr}\,\frac{d}{l}\right)^{0.05}\right]^{\frac{1}{0.95}}}. \tag{2}$$

Thus we have created the possibility of establishing by calculation the temperature difference between a vertically arranged resistive element and the heat-absorbing medium surrounding it. Having the values of $\Delta t$ for all diameters of resistive elements in KRMS standards and knowing their temperature coefficient of resistance, one can then determine the corresponding values of the systematic temperature errors which will occur during measurements at a given power dissipation.

## NOTATION

| | |
|---|---|
| $\alpha$ | is the heat transfer coefficient; |
| $\Delta t$ | is the temperature drop; |
| q | is the heat flux density; |
| $Nu_d$, $Gr_d$, $Ra_d$, Pr | are the Nusselt, Grashof, Rayleigh, and Prandtl numbers; |
| d | is the wire diameter; |
| $l$ | is the length of wire; |
| $U_0$ | is the voltage drop in standard resistor; |
| $U_n$ | is the same on test wire; |
| $R_0$ | is the resistance of standard resistor; |
| $R_n$ | is the same for test wire; |
| $t_n$ | is the temperature of wire; |
| $t_m$ | is the transformer oil temperature; |
| $\lambda$, $\beta$, $\nu$ | are the thermal conductivity, volume expansion coefficient, and kinematic viscosity; |
| g | is the gravity acceleration; |
| P | is the value of scattering power during experiment. |

## LITERATURE CITED


1. G. V. Mchedlidze, Proceedings of the II Republic Scientific and Technical Conference on Metrology [in Russian], Tbilisi Branch, All-Union Scientific-Research Institute of Metrology, Tbilisi (1972).
2. F. J. Wilkins and M. J. Swan, Proc. IEE, 117, No. 4, 841 (1970).
3. A. F. Dunn, IEEE Trans. on IM, IM-15, No. 4, 220 (1966).
4. H. R. Nagendra et al., Trans ASME, Series C, 92, 191 (1970).
5. Instruments for Temperature Measurement and Their Testing [in Russian], Mashgiz, Moscow (1955).